SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

AS
FURTHER MATHEMATICS

Paper 2 – Discrete

Exam Date Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- You must ensure you have the other optional question paper/answer booklet for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete both papers.
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A graph has 5 vertices and 6 edges.

Find the sum of the degrees of the vertices.

Circle your answer.

[1 mark]

10 11 12 15

$2 \times 6 = 12$ ✓

2 A connected planar graph has x vertices and $2x - 4$ edges.

Find the number of faces of the planar graph in terms of x.

Circle your answer.

[1 mark]

$x - 6$ $x - 2$ $6 - x$ $2 - x$

$v - e + f = 2$

$x - (2x - 4) + f = 2$

$f = 2 + x - 4$

$= x - 2$ ✓

3 The function min (a, b) is defined by:

$$\min(a,b) \quad = a, a < b$$
$$\qquad\qquad = b, \text{otherwise}$$

For example, min (7, 2) = 2 and min (–4, 6) = –4.

Gary claims that the binary operation Δ, which is defined as

$$x \Delta y = \min(x, y-3)$$

where x and y are real numbers, is associative as finding the smallest number is not affected by the order of operation.

Disprove Gary's claim.

[2 marks]

if associative, $(x \Delta y) \Delta z = x \Delta (y \Delta z)$
find a counter example, e.g. $(3 \Delta 1) \Delta 2 = -2 \Delta 2$
$= -2$

$3 \Delta (1 \Delta 2) = 3 \Delta -1 = -4$
$-2 \neq -4$ so not associative, we have
disproven Gary's claim

Turn over for the next question

4 A communications company is conducting a feasibility study into the installation of underground television cables between 5 neighbouring districts.

The length of the possible pathways for the television cables between each pair of districts, in miles, is shown in the table.

The pathways all run alongside cycle tracks.

	B	G	H	O	U
	Billinge	**Garswood**	**Haydock**	**Orrell**	**Up Holland**
Billinge	-	2.5	***	4.3	4.8
Garswood	2.5	-	3.1	***	5.9
Haydock	***	3.1	-	6.7	7.8
Orrell	4.3	***	6.7	-	2.1
Up Holland	4.8	5.9	7.8	2.1	-

4 (a) Give a possible reason, in context, why some of the table entries are labelled as ***.

[1 mark]

Sometimes a cable cannot be laid between two districts because of something in the way, eg. a river

4 (b) As part of the feasibility study, Sally, an engineer needs to assess each possible pathway between the districts. To do this, Sally decides to travel along every pathway using a bicycle, starting and finishing in the same district. From past experience, Sally knows that she can travel at an average speed of 12 miles per hour on a bicycle.

Find the minimum time, in minutes, that it will take Sally to cycle along every pathway.

[4 marks]

B, G, H and O are odd nodes, U is even, so we need to find the minimum pair of shortest distances. Work out the shortest distance between each pair: B-G: 2.5 G-O: 6.8
B-O: 4.3 G-H: 3.1
B-H: 5.6 H-O: 6.7

Shortest total that includes all nodes once is B-O, G-H so repeat these → total repeated distance = 4.3+3.1 = 7.4

total = 37.2 + 7.4 = 44.6 miles

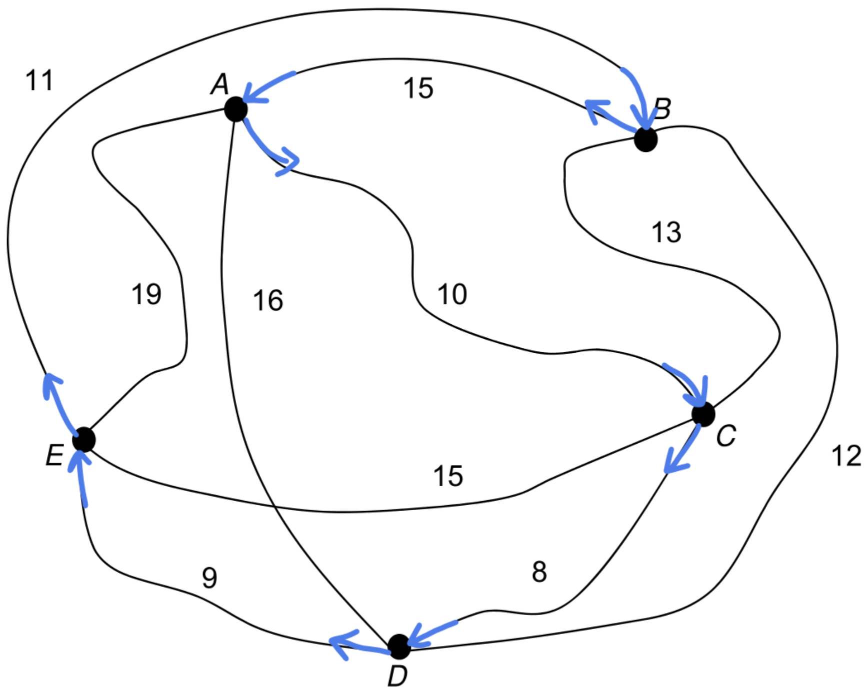
this will take her $\frac{44.6}{12}$ hours $= \frac{44.6}{12} \times 60$ minutes

$= 223$ minutes

Turn over for the next question

5 Charlotte is visiting a city and plans to visit its five monuments: *A*, *B*, *C*, *D* and *E*.

The network shows the time, in minutes, that a typical tourist would take to walk between the monuments on a busy weekday morning.



Charlotte intends to walk from one monument to another until she has visited them all, before returning to her starting place.

5 (a) Use the nearest neighbour algorithm, starting from *A*, to find an upper bound for the minimum time for Charlotte's tour.

[2 marks]

$A \to C \to D \to E \to B \to A$

$10+8+9+11+15=\underline{53}$

5 (b) By deleting vertex *B*, find a lower bound for the minimum time for Charlotte's tour.

[3 marks]

the four paths between the four remaining vertices have length 9, 19, 10, & 8. 19 is the longest so we use the other three: 9+8+10=27.
now we need to connect vertex B, the possible edges have lengths 11, 15, 13 & 12. choose 11 & 12 (EB&ED):
11+12=23
27+23=50

5 (c) Charlotte wants to complete the tour in 52 minutes. Use your answers to parts **(a)** and **(b)** to comment on whether this could be possible.

[1 mark]

the optimal cycle lies between 50 & 53 minutes, but because the lower bound is not a Hamiltonian cycle, we cannot be sure a HC of <53 exists.

5 (d) Charlotte takes 58 minutes to complete the tour. Evaluate your answers to part **(a)** and part **(b)** given this information.

[1 mark]

this is longer than a typical tourist, but she may walk slower or take more breaks than is typical

5 (e) Explain how this model for a typical tourist's tour may not be applicable if the tourist walked between the monuments during the evening.

[1 mark]

the amount of traffic may be less, in which case they could walk quicker & not wait as long to cross roads.

6 Victoria and Albert play a zero-sum game. The game is represented by the following pay-off matrix for Victoria.

		Albert		
	Strategy	***X***	***Y***	***Z***
Victoria	***P***	3	–1	1
	Q	–2	0	1
	R	4	–1	–1

6 (a) Find the play-safe strategies for each player.

[3 marks]

row minima: -1, -2, -1
column maxima: 4, 0, 1
max. of row minima = -1
min. of column maxima = 0
so Albert plays Y, Victoria can play R or P.

6 (b) State, with a reason, the strategy that Albert should never play.

[1 mark]

whatever Victoria plays, Y will always dominate strategy Z,
so he should never play Z

6 (c) (i) Determine an optimal mixed strategy for Victoria.

[5 marks]

R dominates P so she should never play P. She either has to do R or Q. let p be the probability she plays Q, &(1-p) for playing R.

6 (c) (ii) Find the value of the game for Victoria.

[1 mark]

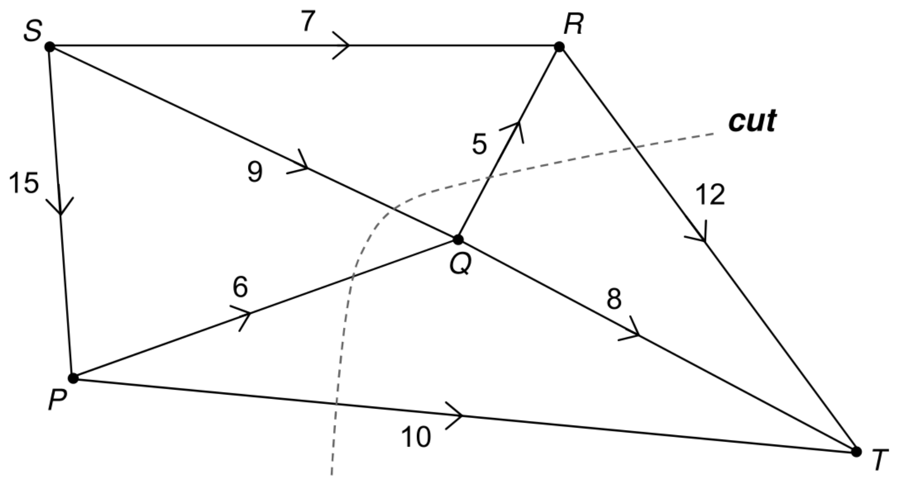
6 (c) (iii) State an assumption that must made in order that your answer for part **(c)(ii)** is the maximum expected pay-off that Victoria can achieve.

[1 mark]

7 The network shows a system of pipes, where S is the source and T is the sink.

The capacity, in litres per second, of each pipe is shown on each arc.

The cut shown in the diagram can be represented as $\{S, P, R\}$, $\{Q, T\}$.



7 (a) Complete the table below to give the value of each of the 8 possible cuts.

[1 mark]

Cut		Value
$\{S\}$	$\{P, Q, R, T\}$	31
$\{S, P\}$	$\{Q, R, T\}$	32
$\{S, Q\}$	$\{P, R, T\}$	
$\{S, R\}$	$\{P, Q, T\}$	
$\{S, P, Q\}$	$\{R, T\}$	30
$\{S, P, R\}$	$\{Q, T\}$	37
$\{S, Q, R\}$	$\{P, T\}$	35
$\{S, P, Q, R\}$	$\{T\}$	30

7 (b) State the value of the maximum flow through the network.

Give a reason for your answer.

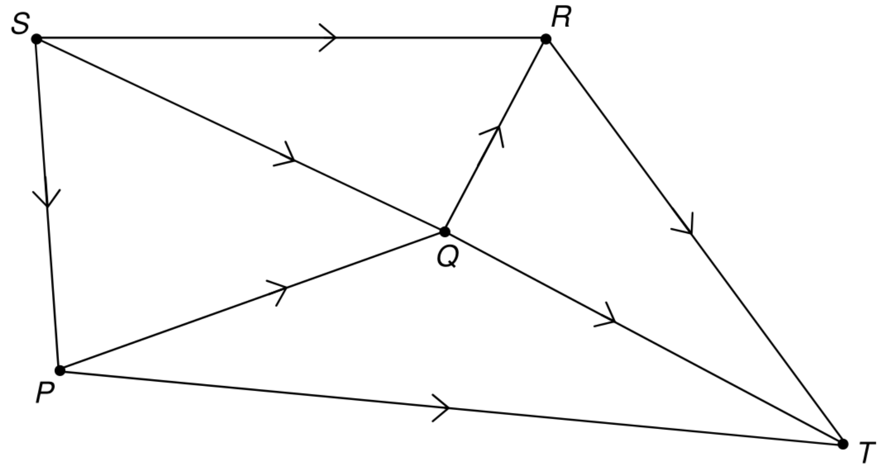
[1 mark]

---

7 (c) Indicate on **Figure 1** a possible flow along each arc, corresponding to the maximum flow through the network.

[2 marks]

Figure 1



8 A family business makes and sells two kinds of kitchen table.

Each pine table takes 6 hours to make and the cost of materials is £30.

Each oak table takes 10 hours to make and the cost of materials is £70.

Each month, the business has 360 hours available for making the tables and £2100 available for the materials.

Each month, the business sells all of its tables to a wholesaler.

The wholesaler specifies that it requires at least 10 oak tables per month and at least as many pine tables as oak tables.

Each pine table sold gives the business a profit of £40 and each oak table sold gives the business a profit of £75.

Use a graphical method to find the number of each type of table the business should make each month, in order to maximise its total profit.

Show clearly how you obtain your answer.

[8 marks]

END OF QUESTIONS